GUT scenarios

and their respective signatures

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Grand Unification

Three Hints :

- Neutrino masses point at new physics around 10^{14} GeV:

$$\frac{1}{M_N} L\, L\, H_u\, H_u \quad : \quad m^{\nu} \sim \frac{(100\ \text{GeV})^2}{M_N} \sim 0.1\ \text{eV} \quad \longrightarrow \quad M_N \sim 10^{14}\ \text{GeV}$$

- The symmetries and the particle content of the standard model point towards Grand Unified Theories as the next step in the unification of all forces.

  Neutrinos are generically massive in GUTs with rank-5 group.

- The gauge couplings of the (supersymmetric) standard model converge between 10^{14} and 10^{16} GeV.

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- We need to detect proton decay!

Grand Unification and Proton Decay

	low-energy Supersymmetry?	
dominant decay	YES	NO
gauge boson dominated $p \longrightarrow e^+\pi^0$		
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- Proton decay mediated by Higgs superfields. $\longrightarrow$ dimension-five operators

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- Dimension-five operators are suppressed or absent.
- Proton decay mediating gauge bosons are absent.

SO(10) GUT in Six Dimensions

[Asaka, Buchmüller, Covi; Hall, Nomura, Okul, Smith 2001]

Consider 6D N=1 SUSY SO(10) compactified on $T^2/\left(\mathbb{Z}_2 \times \mathbb{Z}_2^{\text{PS}} \times \mathbb{Z}_2^{\text{GG}}\right)$.

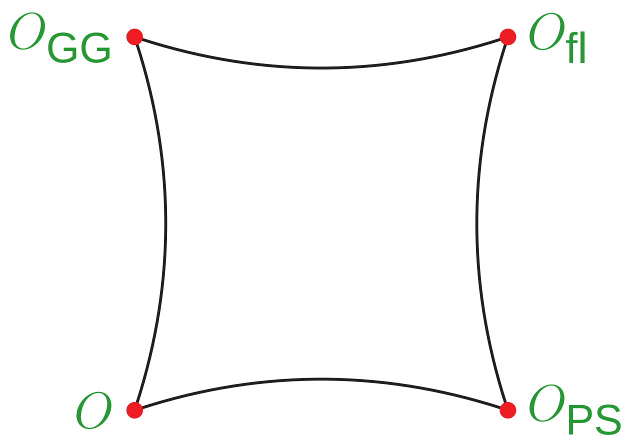
The discrete symmetries break the extended supersymmetry and

$$P_{\text{PS}} : \text{SO(10)} \to \text{SU(4)} \times \text{SU(2)} \times \text{SU(2)}\ , \qquad P_{\text{GG}} : \text{SO(10)} \to \text{SU(5)} \times \text{U(1)}\ .$$

The breaking is localized at different points in the extra dimensions, O, O_{PS} and O_{GG}.

Physical region:

'pillow' with four fixed points (branes).



→ Four fixed points with local gauge groups SO(10), SU(5), flipped SU(5), and G_{PS}.

The unbroken gauge group of the effective 4D theory is given by the intersection of the subgroups at the fixed points, $\text{G}_{\text{SM}} \times \text{U(1)}$.

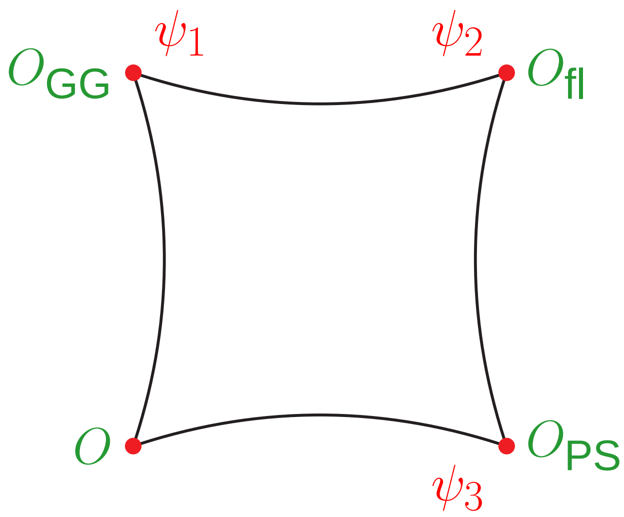
Fermion Masses and Mixing

The three families are located at different branes; they have only diagonal Yukawa couplings with the bulk Higgs fields, direct mixings are exponentially suppressed.

However, they mix with bulk field zero modes without suppression, so the zero modes

$$L_4(\phi) = \begin{pmatrix} \nu_4 \\ e_4 \end{pmatrix}, \qquad L_4^c(\phi^c) = \begin{pmatrix} \nu_4^c \\ e_4^c \end{pmatrix}, \qquad d_4^c(H_5) \qquad d_4(H_6) \ ,$$

act as a (vectorial) fourth generation of down quarks and leptons and mix with the three generations of brane fields.



- Connect hierarchy to the location of the SM generations in the extra space-coordinates;
- the Yukawa couplings satisfy only the GUT symmetry of the local fixed point (local unification);
- the mixing between generations appears due to mixing with bulk fields zero modes with the same quantum numbers.

Neutrino Masses and Mixings

Neutrino phenomenology is fixed in terms of quark masses and mixings.

The light neutrino masses result from the see-saw mechanism, yielding

$$\frac{m_2}{m_3} \sim \frac{m_s^2}{m_b^2}\frac{m_t}{m_c} \sim 0.2 \;, \qquad \frac{m_1}{m_3} \sim \frac{m_d^2}{m_b^2}\frac{m_t}{m_u} \sim 0.2 \;.$$

- The weak hierarchy in the neutrino sector can be traced back to the non-perfect compensation between down and up quark hierarchies.
- The light neutrino mass spectrum is hierarchical with

$$m_1 \lesssim m_2 \sim \sqrt{\Delta m^2_{\text{sol}}} < m_3 \sim \sqrt{\Delta m^2_{\text{atm}}} \;.$$

- Both atmospheric and solar mixing angles are naturally large;

  smallness of the electron mass yields small reactor mixing angle, $\theta_{13} = \mathcal{O}\left(0.1\right)$.

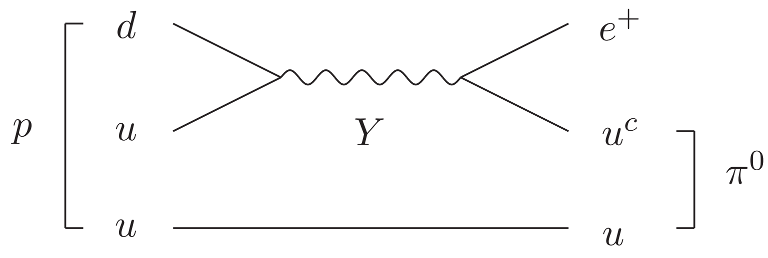
[Buchmüller, Covi, Emmanuel-Costa, SW 2007]

Proton Decay

Dimension-five operators do not appear, since the triplets in H_1 and H_2 do not have a common mass term: [Altarelli, Feruglio; Hall, Nomura 2001]
KK mass comes from the 6D kinetic term and mixes H_i with its N=2 SUSY partner $\widetilde{H}_i$.

ψ_1 at O_{GG} $\rightarrow$ Proton decay via X, Y-boson

$$\mathscr{L}_{\text{eff}} \propto \epsilon^{\alpha\beta\gamma} \left[\overline{e^c}\, \overline{u^c_\alpha}\, Q_\beta\, Q_\gamma - \overline{d^c_\alpha}\, \overline{u^c_\beta}\, Q_\gamma L \right]$$



- Non-universal coupling of gauge bosons:
  Only first generation's weak eigenstates couple to X, Y.

- Coupling: KK tower

$$\frac{1}{(M_V^{\text{eff}})^2} = 2 \sum_{n,m=0}^{\infty} \frac{1}{\frac{(2n+1)^2}{R_5^2} + \frac{(2m)^2}{R_6^2}} \xrightarrow{R_5=R_6=\frac{1}{M_c}} \boxed{\frac{\pi}{4M_c^2}\left(\ln\left(\frac{M_*}{M_c}\right) + 2.3\right)}$$

- There are (small) corrections from derivative brane operators.

Branching Ratios

decay mode	6D SO(10) *(simplest scen.)*	4D model
$e^+\,\pi^0$	75%	54%
$\mu^+\,\pi^0$	4%	< 1%
$\bar\nu\,\pi^+$	19%	27%
$e^+\,K^0$	1%	< 1%
$\mu^+\,K^0$	< 1%	18%
$\bar\nu\,K^+$	< 1%	< 1%
$e^+\,\eta$	< 1%	< 1%
$\mu^+\,\eta$	< 1%	< 1%

4D model :
SU(5) with lopsided family structure

Suppression of $p \to \mu^+ K^0$:

- Absence of 2^{nd} and 3^{rd} generation;
- small (12) component in rotation matrices U_R^d and U_L^e.

$\to$ doubly Cabibbo suppressed!

Possibility to distinguish between conventional and orbifold GUTs !

Derive upper limit for BR $p \to \mu^+ K^0$: $\dfrac{\Gamma(p \to \mu^+ K^0)}{\Gamma(p \to e^+ \pi^0)} \lesssim 5\,\%$ $\leftrightarrow$ 4D: $33\,\%$

$$\Gamma(p \to e^+\pi^0) \simeq \left(\frac{9\times10^{15}\ \text{GeV}}{M_c}\right)^4 \left(5.3\times10^{33}\ \text{yrs}\right)^{-1}$$

[Buchmüller, Covi, Emmanuel-Costa, SW 2004]

Trinification

$\boxed{\mathrm{G}_{\mathrm{TR}} = \mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathbb{Z}_3}$

[Achiman, Stech 1978; de Rújula, Georgi, Glashow 1984;
Babu, He, Pakvasa 1986]

- $\mathbb{Z}_3$ guarantess that gauge couplings conincide at M_U;
- no need for adjoint Higgs fields;
- up to five light Higgs doublets in its minimal version.

$\longrightarrow$ Gauge-coupling unification may result at $M_{\mathrm{U}} \simeq 10^{14}$ GeV without supersymmetry.

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Fermions: $(3, \bar{3}, 1) \oplus (\bar{3}, 1, 3) \oplus (1, 3, \bar{3}) \equiv \psi_{\mathsf{Q}} \oplus \psi_{\mathsf{Q}^c} \oplus \psi_{\mathsf{L}}$

$$\psi_{\mathsf{Q}} = \begin{pmatrix} (-d \quad u) & B \end{pmatrix}, \qquad \psi_{\mathsf{Q}^c} = \begin{pmatrix} \mathscr{D}^c \\ u^c \\ \mathscr{B}^c \end{pmatrix}, \qquad \psi_{\mathsf{L}} = \begin{pmatrix} (\mathscr{E}) & (E^c) & (\mathscr{L}) \\ \mathscr{N}_1 & e^c & \mathscr{N}_2 \end{pmatrix}.$$

In addition to the 15 SM fermions, there are 12 new fermions:

- one vector-like down quark and lepton doublet ($5 + \bar{5}$ of SU(5));
- one sterile (i.e., $B - L = 0$) neutrino.

Breaking of G_{TR}

Breaking by a pair of $\Phi_{\mathrm{L}}\ (1,3,\bar{3}) = \begin{pmatrix} (\phi_1) & (\phi_2) & (\phi_3) \\ S_1 & S_2 & S_3 \end{pmatrix}$ with

$$\boxed{\langle \Phi_{\mathrm{L}}^1 \rangle = \left(\begin{pmatrix} u_1 \\ 0 \end{pmatrix} \ \begin{pmatrix} 0 \\ u_2 \end{pmatrix} \ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \atop \ \ 0 \qquad\quad 0 \qquad\quad v_1 \right) \text{ and } \langle \Phi_{\mathrm{L}}^2 \rangle = \left(\begin{pmatrix} n_1 \\ 0 \end{pmatrix} \ \begin{pmatrix} 0 \\ n_2 \end{pmatrix} \ \begin{pmatrix} n_3 \\ 0 \end{pmatrix} \atop \ \ v_2 \qquad\quad 0 \qquad\quad 0 \right)}$$

For simplicity, we choose $n_{1,2,3} = 0$ here.

Additional charged fermions become heavy when G_{TR} is broken to G_{SM} (vevs $v_{1,2}$).

Light fermion masses: $m_u = g_1 u_2\,,$ $\quad m_d \simeq g_1 u_1\, s_\alpha\,,$ $\quad \tan\alpha = \frac{g_1 v_1}{g_2 v_2}\,,$

$m_e \simeq h_1 u_1\, s_\beta$ $\quad \tan\beta = \frac{h_1 v_1}{h_2 v_2}$

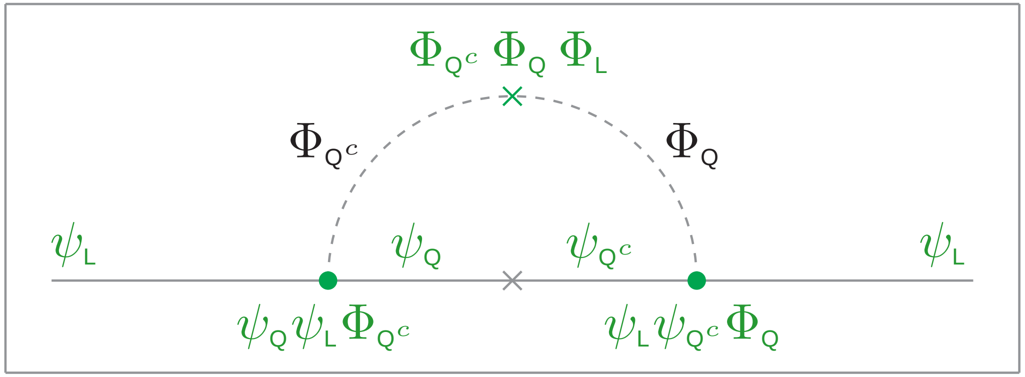
No relation between the masses of the quarks and leptons; the minimal model is sufficient to describe the masses of the quarks and charged leptons.

Radiative Seesaw Mechanism

At tree level, the model yields an active Dirac neutrino at the electroweak scale and a sterile Majorana neutrinoat the eV scale; however, this is corrected by one-loop diagrams with color-charged Higgs bosons:

$\Phi_{\mathsf{Q}}\ (3, \bar{3}, 1)$

$\Phi_{\mathsf{Q}^c}\ (\bar{3}, 1, 3)$



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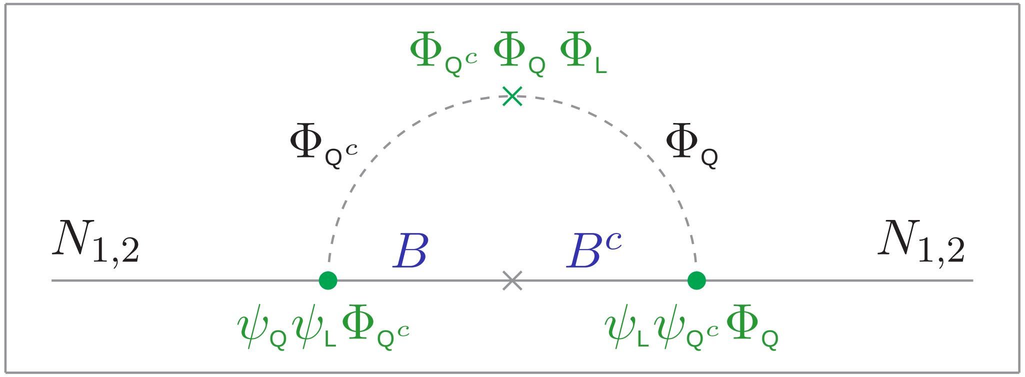


This mechanism is absent in models with low-energy supersymmetry.

$\longrightarrow$ mass matrix for neutrinos (ν, N_1, N_2)

$$M_N \simeq \begin{pmatrix} 0 & -h_1 u_1 & 0 \\ -h_1 u_1 & s_{\alpha-\beta} c_\beta \, g^2 F_q(B) & (s_{2\beta} s_\alpha - c_\alpha)\, g^2 F_q(B) \\ 0 & (s_{2\beta} s_\alpha - c_\alpha)\, g^2 F_q(B) & c_{\alpha-\beta} s_\beta \, g^2 F_q(B) \end{pmatrix}, \qquad F_q(q) \propto m_q$$

(loop integral)

sterile neutrinos obtain masses $\lambda_N \sim F_q(B) \sim \mathcal{O}(M_U)$,

active neutrino is light, $\lambda_\nu \sim \dfrac{(h_1 u_1)^2}{g^2 F_q(B)} \simeq 0.1$ eV !

Proton Decay

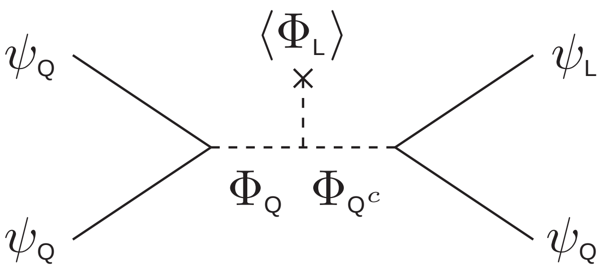
Quarks and leptons in different multiplets. $\longrightarrow$ No proton decay via gauge bosons.

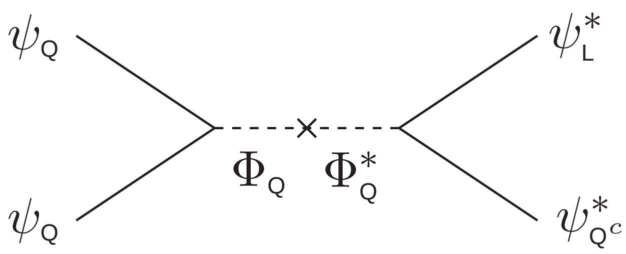
Instead, proton decay is mediated by Higgs fields Φ_{Q^c} and Φ_{Q}.

These dimension-six operators are suppressed by small Yukawa couplings,

$$\left\{g\,U_\beta\,h\,QQQL - g\,U_\alpha^\top\,h\,d^c u^c e^c u^c\right\} - \left\{g^* h\,QQe^{c*}u^{c*} - g\,U_\beta\,U_\alpha^\dagger\,h^*\,d^{c*}u^{c*}QL\right\}$$

$\longrightarrow$ Flavor non-diagonal decay dominant, in particular $p \to \bar{\nu}K^+$.





$\longrightarrow$ Estimated lifetime: $\tau \simeq \left(\frac{1}{g\,h}\right)^2 \times 10^{28}$ years $\quad \Rightarrow \quad g\,h \lesssim 10^{-3}$.

The decay width of $p \to \bar{\nu}K^+$ is close to the experimental limit;

$p \to \bar{\nu}\pi^+$ and $p \to \mu^+K^0$ should be visible.

[Sayre, Willenbrock, SW 2006]

Summary

GUTs provide a beautiful framework for theories beyond the standard model.

The detection of proton decay, together with the (non-)obersvation of weak-scale supersymmetry will give us a strong hint at the underlying GUT scenario.

Orbifolds offer an elegant way of GUT symmetry breaking ;
split multiplets are simple explanation of doublet-triplet splitting.

- We have studied a simple SO(10) model, where the SM generations are brane fields on different locations and mixing arises due to mixing with bulk split multiplets.
- Neutrino masses and mixings are determined by the mismatch of up and down-quark hierarchies.
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The trinified model is a viable candidate for non-supersymmetric unification .

- No need to introduce intermediate scales, additional Higgs fields, or higher-dimensional operators.
- Proton decay is mediated by color-charged Higgs bosons.